



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE ANGLE-SUM ACCORDING TO PLAYFAIR.

By Professor JOHN N. LYLE, Ph. D., Westminster College, Fulton, Mo.

In Playfair's Euclid, pages 295 and 296, there is given a short method for finding the angle-sum of a rectilinear triangle.

As the soundness of this method has been called in question by the Hyper-Space theorists, it is incumbent upon teachers of geometry to examine both the method itself and the criticisms to which it has been subjected.

John Playfair in treating of the angle-sum says—"It is of importance in explaining the Elements of Science, to connect truths by the shortest chain possible ; and till that is done, we can never consider them as being placed in their *natural order*.

The reasoning in the first of the following propositions is so simple, that it seems hardly susceptible of abbreviation, and it has the advantage of connecting immediately two truths so much alike, that one might conclude, even from the bare enunciations, that they are but different cases of the same general theorem, viz., That all the angles about a point, and all the exterior angles of any rectilinear figure, are constantly of the same magnitude, and equal to four right angles.

DEFINITION.

If, while one extremity of a straight line remains fixed at A , the line itself turns about that point from the position AB to the position AC , it is said to describe the angle BAC contained by the lines AB and AC .



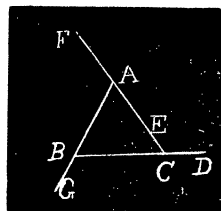
Corollary. If a line turn about a point from the position AC till it come into the position AC again, it describes angles which are together equal to four right angles. This is evident from the second corollary to the fifteenth, 1.

PROPOSITION I.

All the exterior angles of any rectilinear figure are together equal to four right angles.

1. Let the rectilinear figure be the triangle ABC , of which the exterior angles are DCA , FAB , GBC ; these angles are together equal to four right angles.

Let the line CD , placed in the direction BC produced, turn about the point C till it coincide with CE , a part of the side CA , and have described the exterior angle DCE or DCA .



Let it then be carried along the line CA , till it be in the position AF , that is, in the direction of CA produced, and the point A remaining fixed, let it turn about A till it describe the angle FAB , and coincide with a part of the line AB .

Let it next be carried along AB until it come into the position BG , and by turning about B , let it describe the angle GBC so as to coincide with a part of BC .

Lastly, let it be carried along BC till it coincide with CD its first position.

Then, because the line CD has turned about one of its extremities till it has come into the position CD again, it has by the corollary to the above definition described angles which are together equal to four right angles; but the angles which it has described are the three exterior angles of the triangle ABC , therefore the exterior angles of the triangle ABC are equal to four right angles.

2. If the rectilineal figure have any number of sides, the proposition is demonstrated just as in the case of a triangle. Therefore all the exterior angles of any rectilineal figure are together equal to four right angles.

Corollary 1. Hence, all the interior angles of any triangle are equal to two right angles. For all the angles of the triangle, both exterior and interior, are equal to six right angles, and the exterior being equal to four right angles, the interior are equal to two right angles."

In this demonstration of the angle sum Playfair evidently regards the method employed by him as legitimate, simple, direct and brief.

The Riemannian division of the Hyper-Space theorists assumes that a plane is the surface of an immense sphere, and that straight lines are curves that come back to their starting points, and, hence, raises the objection that such lines can not be slid along and then rotated as Playfair's demonstration requires.

This objection of the Riemannian School obviously rests on the false bottom that a plane is a spherical surface and that straight lines are curves. The foundation being insecure, that which is built thereon can not stand. The objection obliterates the distinction between spherical geometry and plane geometry.

If it be true that a plane is perfectly flat and that straight lines are devoid of curvature, the objection that we are considering is seen to have no force.

The Riemannian theorists tell us that for ought they know straight lines may be curves. They begin by doubting the truth of Euclid's second postulate—"That a terminated straight line may be produced to any length in a straight line"—and his Proposition XXXII, Book I. They are believers, also, as well as doubters. They believe that the angle-sum of a rectilineal triangle is greater than two right angles. They believe that if a straight line be extended it will ultimately return to the starting point.

The Euclidean geometers doubt these articles of Riemannian faith, and believe that the angle-sum of a rectilineal triangle is equal to two right angles, and that the longer a straight line is the further apart are its ends.

The Riemannians doubt what the Euclidean believe and believe what the Euclidean doubt. Those who undertake to teach both of these doctrines that contradict each other have failed to reckon with the logical laws of non-contradiction and excluded middle. We notice further that the Riemannian objection to Playfair's demonstration is in conflict with Proposition I of Lobatschewsky's

Theory of Parallels. Says the Russian Pangeometer—"A straight line fits upon itself in all its positions. By this I mean that during the revolution of the surface containing it the straight line does not change its place, if it goes through two unmoving points in the surface: (*i. e.*, if we turn the surface containing it about two points of the line, the line does not move)." These statements can not be made of any arc of any circle, and, hence, can not be made of Riemannian straight lines that are assumed to have constant positive curvature. What Lobatschewsky says respecting the straight line in his theorem I is inconsistent, of course, with his doctrine that the angle-sum is less than two right angles. But we are not quoting Lobatschewsky now to show that his theory is inconsistent with itself, but with that of Riemann.

Another objection to Playfair's demonstration is that a triangle drawn on a blackboard is not bounded by lines perfectly straight, since the surface of the board is uneven.

This objection does not hold against the triangle whose vertices are the three points A , B , and C in space and whose sides are destitute of curvature.

The geometer, whether he proceeds analytically or synthetically, naturally regards space as extending beyond himself on all sides without bounds, and between any two points A and B located therein he can draw an absolutely straight line with his mind, although he may be unable to do so with his hand. Some metaphysicians doubt these facts. What is it that they have not doubted? The function of a metaphysician, however, is to explain facts, not to doubt or discredit them.

Three points A , B , and C not in the same straight line may be located in trinally extended objective space and connected by the straight lines AB , AC , and BC . Hence, a rectilineal triangle in objective space is possible. When we say rectilineal triangle we do not mean a bogus triangle with wrinkled sides, but a genuine triangle with straight sides. When we say straight sides we do not mean wrinkled sides. The rectilineal triangle ABC of the geometer is perfect. His ability to cognize such a triangle is shown in the fact that he does cognize it. This fact, too, has been doubted. What a wonderful endowment that must be that enables man to people space with faultlessly perfect forms! This lofty power of intelligence in man, nay even his own doubt respecting it, differentiates him from the lower animals.